Translating figures in coordinate space

In this lesson we'll look at how the translation of a figure in a coordinate plane determines where it's located.

A **translation** is a transformation that moves a figure in a coordinate plane from one location to another. In this way a translation can be thought of as a slide with no rotating. The slide won't change the shape or size of the figure, and with no rotation, the orientation won't change either.

The pre-image and image

A transformation of a shape always starts with the **pre-image**. The pre-image is usually labeled with capital letters. After the pre-image is translated, the final version of the figure is called the **image**, which is usually labeled with the same capital letters, plus the prime symbol, $'$. So if figure $ABCD$ is translated, its image becomes figure $A'B'C'D'$.

The image and pre-image are always congruent to each other, because the translation never changes the angles measures or side lengths of the figure.

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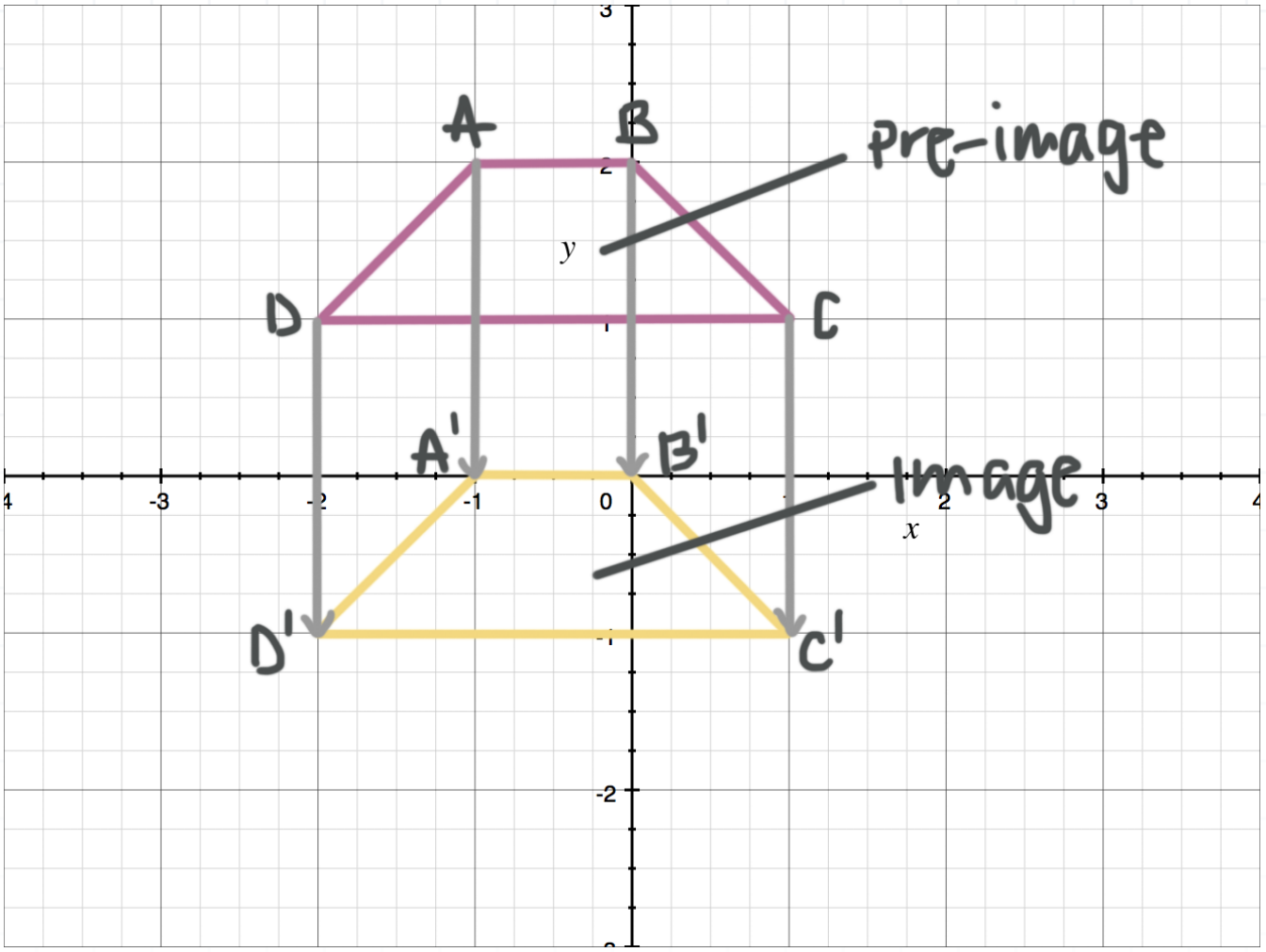

Translation notation

A translation can be vertical, horizontal, or both. Regardless of the direction, a translation can be written in mathematical notation.

A translation 3 units to the **left**: $T(x, y) = (x - 3, y)$

A translation 2 units to the **right**: $T(x, y) = (x + 2, y)$

A translation 4 units **down**: $T(x, y) = (x, y - 4)$

A translation 1 unit **up**: $T(x, y) = (x, y + 1)$

In this triangle translation, the pre-image on the left is translated 2 units to the right, $T(x, y) = (x + 2, y)$, in order to get to the image on the right.

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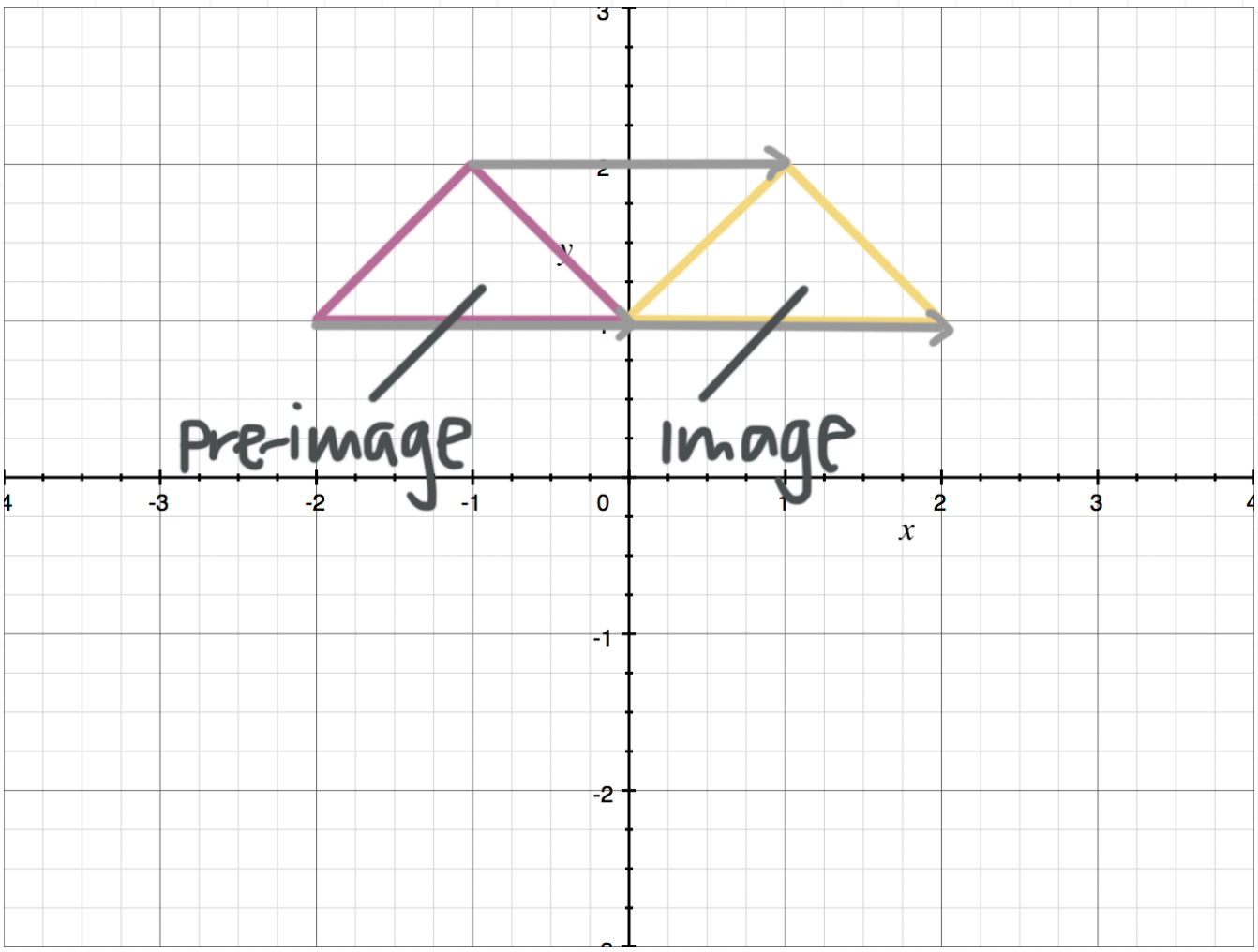

In this triangle translation, the pre-image at the top is translated 4 units down, $T(x, y) = (x, y - 4)$, in order to get to the image at the bottom.

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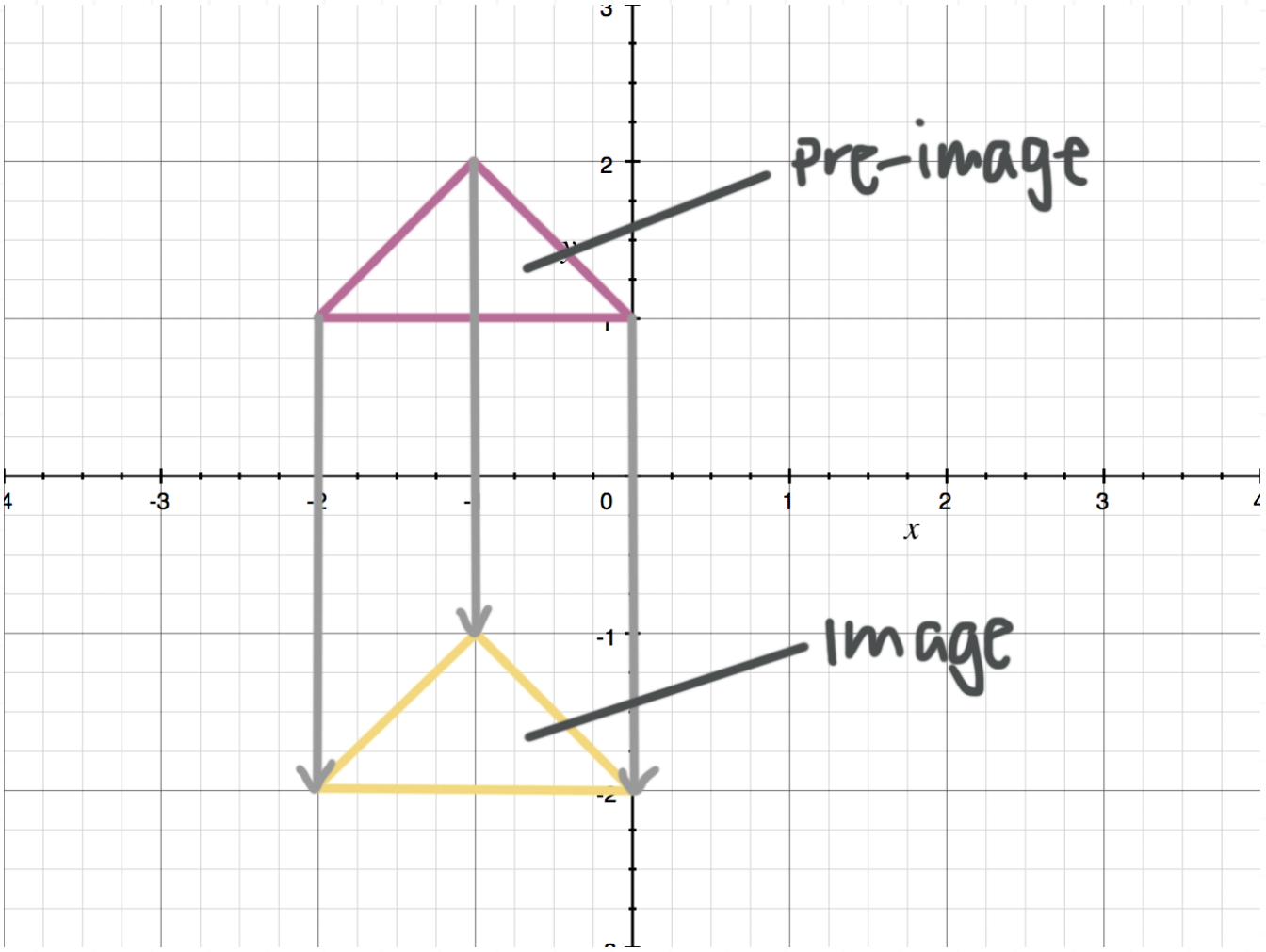

Two translations together

We said that we can do a horizontal and vertical translation at the same time. If we want to translate a figure 2 units to the right and 4 units down, then the translation is

$$T(x, y) = (x + 2, y - 4)$$

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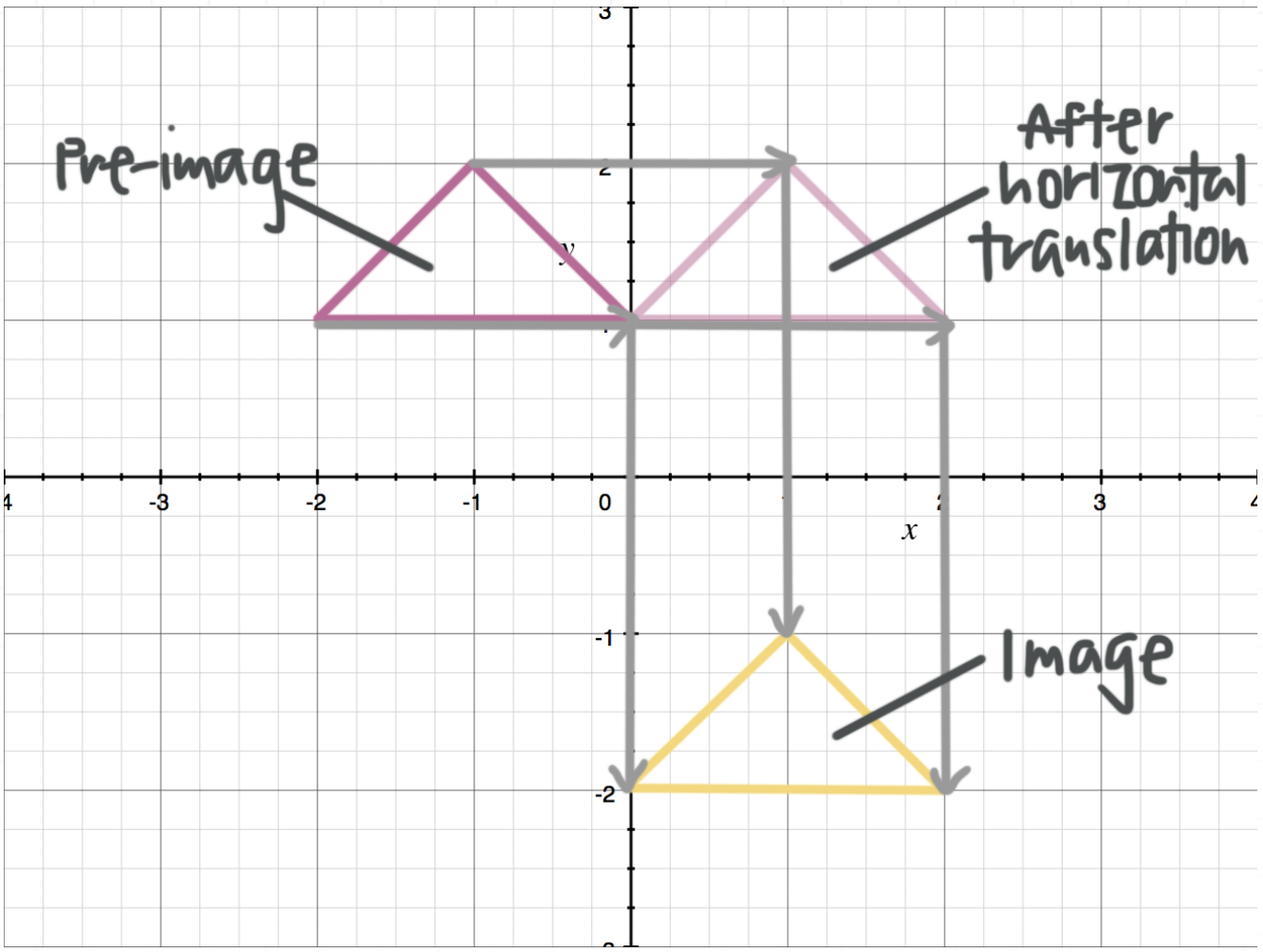

Let's start by working through an example.

Example

If rectangle $ABCD$ undergoes the translation described by $T(x, y) = (x - 2, y + 3)$, where will point D' be located?

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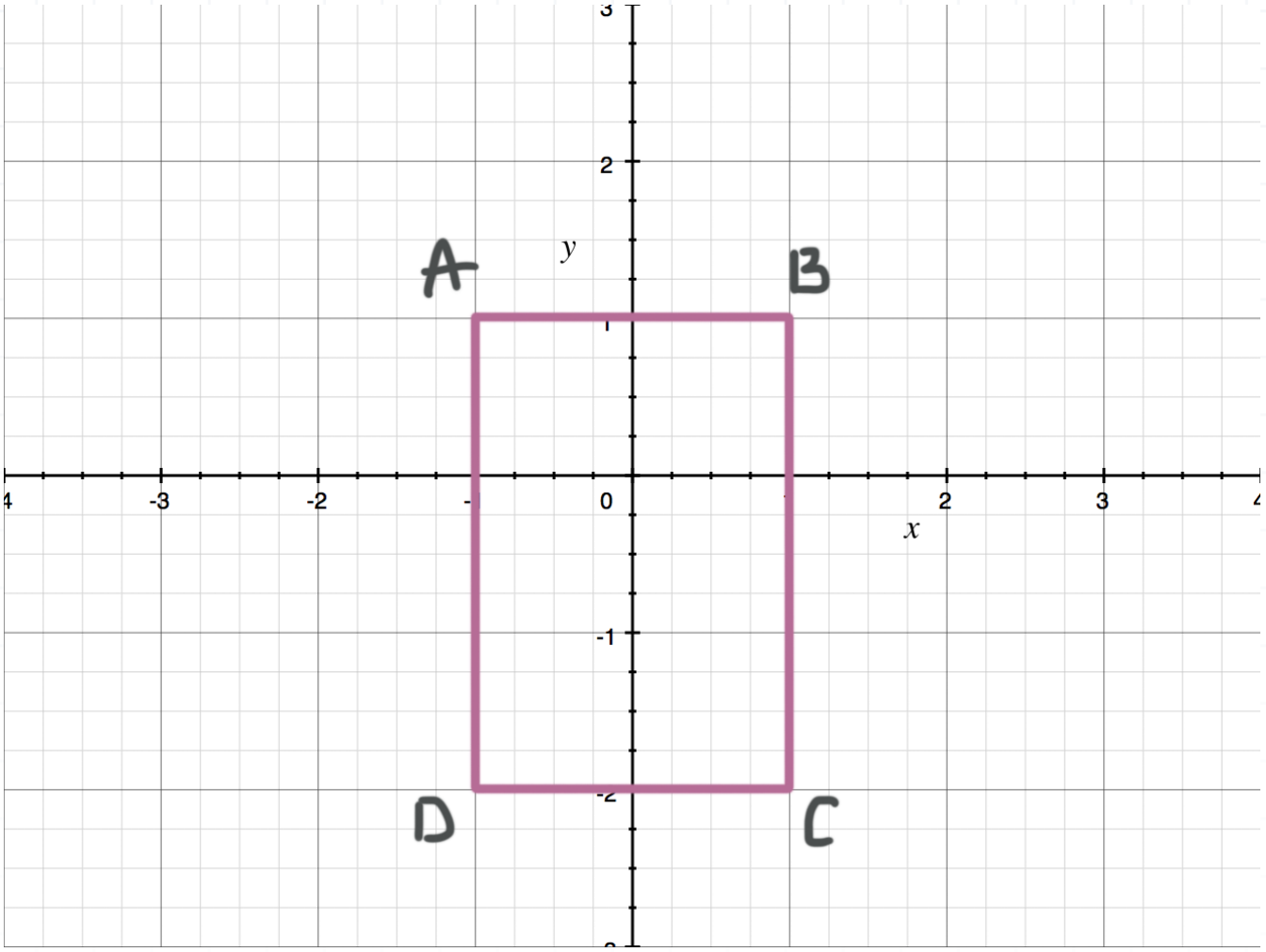

The translation is $T(x, y) = (x - 2, y + 3)$. The $x - 2$ tells you that in the new set of points, each x-value will now be 2 less than it was in the old set of points and each y-value will now be 3 more than it was with the old set of points.

In other words, the figure will be located 2 units to the left and 3 units above the old set of points. The original point D was $(-1, -2)$, so the new point D' will be

$$T(-1, -2) = (-1 - 2, -2 + 3) = (-3, 1)$$

So $D' = (-3, 1)$.

Let's try another translation problem.

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Example

If a translation moves A to A', write a rule for the translation if $A = (-3,7)$ and $A' = (5, -2)$.

Let's look at what happens to each coordinate.

The x-value: $-3 \to 5$

This means we added 8 because $-3 + 8 = 5$.

For the y-value: $7 \to -2$

This means we subtracted 9 because $7 - 9 = -2$.

Now we can put this all together to write the rule for the translation.

$$T(x, y) = (x + 8, y - 9)$$

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